Revisão das Técnicas para a Assimilação do Espectro Direcional Bi-Dimensional em Modelos de Ondas

A Review of the Techniques for the Assimilation of the Two-Dimensional Directional Spectrum into Wave Models

Nelson Violante-Carvalho
Rio de Janeiro State University UERJ
Department of Oceanography
violantecarvalho@yahoo.co.uk
A. V. C. Ramos
Rio de Janeiro Federal University UFRJ
Department of Mathematics

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Abstract: Several meteorological centers are investigating methods on how to use the new information retrieved from spaceborn Synthetic Aperture Radar (SAR) measurements in order to improve the wave forecasting. With this picture in mind, and focusing mainly on the problem of the full directional spectrum, the theory of wave data assimilation techniques is reviewed. The present work aims as well to describe in some detail the three techniques used so far in the assimilation of the two dimensional spectrum, that is the Optimal Interpolation Scheme, the Adjoint Technique and the Green's Function Method.

Key words: Assimilation of the two dimensional wave spectrum; Synthetic; Aperture Radar (SAR); WAM Wave Model.

1 Introduction

The aim of data assimilation is to take advantage of the available observations by introducing them into modeling procedures, in such a way that the forcing or the initial conditions are improved giving better predictions with the model. Data assimilation methods have been used for over four decades in meteorological models with the objective of improving the forecasting by making use of the widely available network of meteo-stations all over the world. Wave modelers, in contrast, have put off such an approach mainly due to the sparseness of wave observations. However after the advent of satellite oceanography this picture has changed and, in the particular case of wind waves, since the 1990's measurements of significant wave heights from altimeters became available and have been assimilated by several weather centers. Nevertheless the exercise of distributing the energy averaged over frequency and direction from wave height measurements over the whole two-dimensional spectrum requires several assumptions to be made, specially about the separation of wind sea and swell (THOMAS, 1988; JANSSEN and BIDLOT, 2001; VIOLANTE-CARVALHO et al., 2004).

But with the advent of SAR measurements and with the better understanding of the imaging processes, retrieval algorithms have been proposed and the full directional spectra extracted from SAR images are now available in quasi-real time with global coverage (VIOLANTE-CARVALHO et al., 2005). It is recognized that the assimilation of wave observations can improve both the present sea state and, in the case of swell, the forecast of the models (see for example LIONELLO et al., 1995; VOORRIPS et al., 1997; BREIVIK et al., 1998; DUNLAP et al., 1998).

Data assimilation seeks to improve the forecasting introducing available observations into the modeling procedures in order to minimize the differences between model estimates and measurements. Both model and data are assumed to contain errors, which must be taken into consideration during the assimilation procedure. The assimilation of scatter meter, altimeter and SAR data can be applied in a combined wind and wave data assimilation procedure to improve the modeled data, and its difference from the observed data should be smaller than before. As the number of observations to be assimilated is inevitably less than the model grid the data inserted at one grid point must be distributed over neighboring points. To avoid discontinuities the information should be interpolated using either sequential or variational methods.

Sequential methods (also known as kinematic) are time independent assimilations because they make corrections only at the time when an observation is available, in general over a synoptic interval of 6 hours. The strategy is to run the model forward in time, stopping at intervals to assimilate the available observations and then continuing the model run with the corrected state. Therefore winds are updated only locally, although waves in a particular grid point are the result of winds acting in a large area over a large period of time. These methods are computationally cheaper than variational methods, which make them particularly fit for operational use. Some examples of Sequential methods are the Optimal Interpolation (LIONELLO et al., 1992; HASSELMANN et al., 1997; VOORRIPS et al., 1997), the Kalman filter (Voorrips et al.,1999) and Successive Corrections (BREIVIK et al., 1998). Due to its widespread use and simplicity, compared to the others sequential methods (the Kalman filter and the Successive Corrections), our focus here is to provide a comprehensive description of the Optimal Interpolation method.

The Optimal Interpolation method (OI) is the most commonly used sequential method and is implemented operationally at several weather forecast centers in the world, using so far only significant wave heights (SWH) derived from altimeters. In the assimilation of altimeter wave heights some ad hoc assumptions are imposed on the distribution of the energy between wind sea and swell, which are treated separately as in second-generation wave models. Thus one of the most powerful features of third-generation wave models such as WAM is neglected, that is the spectrum has no prescribed form and is free to respond to the source functions. This problem arises because a single point wave height measurement has to be distributed over the whole two-dimensional spectrum, a restriction that no longer applies to the assimilation of retrieved SAR wave spectra.

Variational (or dynamical) are time dependent methods which take the model dynamics into account but have a much higher computational cost compared to sequential methods. A best estimation is obtained through the minimization of a cost function which is dependent on some control variables, generally the wind input.

Observations over different time levels are considered in contrast to the single time level scheme used in Sequential methods. Hence it is possible to correct the wind field that generated a wave component at a time preceding the available observations. A swell generated by a distant storm acts over a large area and the method needs to compute the dynamical regime to track its position back in space and time. So the best model solution not only fits the data available but also is consistent with the constraints of the model. Examples of applications in wave data assimilation are the Adjoint Model (DE LAS HERAS, 1994; HERSBACH, 1998) and the Green's Function Method (BAUER et al., 1996; 1997).

An optimal interpolation method was developed for the WAM model and is operational at ECMWF since the early 1990's. Likewise, a variational data assimilation scheme has been recently implemented (2004) for use with NCEP operational wave model, the NOAA WAVEWATCH-III. In both situations, assimilating only SWH derived from altimeters and some buoys, mostly by the US coast. More information about assimilation of altimeter wave heights is described in Komen et al. (1994, chap. 6). A comprehensive description of assimilation schemes is also presented in de las Heras (1994), while the main purpose of the present work is to review the state of the art of techniques for assimilation of the two dimensional wave spectrum, for instance extracted from SAR images or from buoy measurements.

The structure of the paper is as follows. In section 2 the main aspects of the theory of wave data assimilation are presented. Sections 3, 4 and 5 discuss in more details, respectively, the three most applied wave data assimilation techniques: Optimal Interpolation, Adjoint Model and Green's Function. The final remarks are presented in section 6.

2 Theoretical Basis of Wave Data Assimilation

The evolution of wave energy as a function of frequency, direction, position and time E (f, θ, **r**, t)is represented by the energy balance equation (KOMEN et al., 1994) which for deep water reads

$$\frac{D}{D_t}E = \frac{\partial E}{\partial t} + c_g.\nabla E = S_{in} + S_{nl} + S_{ds} \tag{1}$$

where cg is the group velocity and the right hand side of (1) represents the source and sink terms due, respectively, to wind input, nonlinear interactions and white-capping dissipation. However it is more convenient for data assimilation purposes to rewrite (1) in a matrix form where a set of state variables is forced by a set of control variables (KOMEN et al., 1994):

$$x_{t+1} = F(x_t + u_t) \tag{2}$$

where a state vector xt is the wave energy E at each direction, frequency and grid point at time t and a control vector ut, in general the wind speed, is also defined over each point at time t. The nonlinear functional F represents the physics of the wave model and must be linearized in order to describe how a perturbation in the control vector is dynamically represented by a perturbation in the state vector. Hence performing a Taylor expansion of (2) and retaining only the terms up to the first-order the wave energy balance equation can be represented as

$$x_{t+1} = x_t + \left[\frac{\partial F}{\partial x_t}\right]\delta.x_t + \left[\frac{\partial F}{\partial u_t}\right]\delta.u_t \tag{3}$$

Therefore the nonlinear energy balance equation (1) is rewritten in matrix form (2) and its linear representation (3) describes how perturbations in the state vector (the wave energy E) are determined by perturbations in the wind field, which is the way that variational methods can track back a swell component both in time and in space.

Deviations between observations and model data are used to obtain the best estimate of the wave model. In order to minimize such diferences, the wave model output is modified by adjusting its control variables—the initial conditions or the wind field. The best solution is the one which minimizes a cost function that consists basically of quadratic diferences between observed and modeled data (respectively d^o and d). Considering the probability distribution P(d - d_o; c) given a set of control variables c and assuming that the distribution of the data error representing the model is Gaussian around its maximum (which is reasonable using the Central Limit Theorem) it follows that:

$$P(d-d^0) \cong \exp(-\frac{1}{2}(d-d^0)^2) \tag{4}$$

The maximum of P corresponds to the minimum of the exponent, which means that the maximum probability or most likely state is associated with the minimum of the cost function J, which in matrix notation reads

$$J = (d-d^0)^T M(d-d^0) \tag{5}$$

where M is the expected variance in the model/data error. The calculation of such matrices requires long term statistics of the error covariance of the observations and predictions. Because the true states are not known in most cases, empirical relations are used to approximate M. Diferent weights of the error covariance matrix M are associated with the corrections depending on the distance between the model and observation locations, instrumental errors and model errors. In addition the cost function (5) can be written in a more general form adding terms penalizing diferences between any a priori information available.

The goal of any data assimilation scheme is to minimize the cost function J setting the control variables in order to find the values that yield the minimum diference between modeled data and observations. In most wave data assimilation studies the control variables have been defined as the wind field, although any other parameter that might influence the state vector could be used, as for example, the superficial current field or the initial wave field. The minimization of the cost function J involves the inversion of (5) using the linearized wave model equations (3). However this becomes a non-trivial exerercise due to the size of the state vector, with dimension of the order of 107 in the case of global wave models. Therefore one seeks the minimization of (5) searching for the maximum efficiency and the minimum computational cost.

When all available observations are used for the minimization of the cost function the assimilation scheme is denominated a variational method. In this case the inversion of (5) will require a time-dependency of the wave model equations since observations at a later time t will have to be related to the wave state at some previous time t - 1.

This approach allows the correction of the wind field at some point and time away from the observations, for example tracking back a swell component, but evidently with a high computational cost. On the other hand if only the observations at a single time level are used for the minimization of the cost function the approach is denominated a sequential method. Much simpler and computationaly cheaper this method permits the correction only of the wind associated with the wind sea, and hence only locally. In the following sections we will describe in more details the characteristics of sequential and variational methods and their applications in wave data assimilation studies.

3 The Optimal Interpolation Scheme

The Optimal Interpolation Scheme (OI) is the widest used method for wave data assimilation due especially to its simplicity and low computational cost compared with other techniques. In this scheme the available SAR spectral information is spread over the grid points using statistical interpolation techniques without taking into account the model constraints. The assimilation is performed in two steps. First a best-guess or analyzed field is calculated by Optimal Interpolation and then the corrections applied to the wind sea part of the spectrum are used to correct the local wind.

The analyzed or best estimate value x = (x_i) of the true state vector $x^t = (x_i^t)$ is a linear combination of the model first-guess vector $x^f = (x_i^f)$ obtained from a previous run and the weighted errors between the observed data d_o and the corresponding first-guess values d_f computed from the model:

$$x^f = x_i^f + \sum_{j=1}^{n_{obs}} W_{ij}(d_j^0 - d_j^f) \qquad (6)$$

where i represents each component of the analyzed field, j the component of every observation and n_{obs} denotes the number of observations. W_{ij} are the interpolation weights determined by the minimization of the mean square error between the true state vector and its best estimation

$$J = < (x - x^t)^2 > \qquad (7)$$

This cost function (equation 7) is minimized to obtain the interpolation weight W_{ij} (angle brackets meaning mean values over a large number of realizations). This yields that W_{ij} is a function of the covariance error matrices of the observations and the first-guesses (Komen et al., 1994; Hasselmann et al., 1997). The problem that arises is the computation of these matrices, since long term statistics are needed in order to compare the model predictions with observations. In general empirical relations are used to overcome this problem, and in Voorrips et al. (1997) 2 years of comparison of model results with buoy data are used to determine more refined matrices.

The analyzed data are the result of the Optimal Interpolation scheme (6) and can be applied to diferent types of data, for instance wave heights from altimeters,buoy data and two-dimensional spectra retrieved from SAR images (see for example the description of algorithms for the retrieval of SAR spectra in Hasselmann and Hasselmann, 1991; Krogstad et al., 1994; Hasselmann et al., 1996; Mastenbroek and de Valk, 2000). In Hasselmann et al. (1997) the first step in the assimilation procedure is the optimal interpolation of the two-dimensional SAR wave mode spectrum obtained every 30 s or 200 km along the satellite track. Seeking operational e ciency the number of variables involved in the problem is reduced by partitioning the 2-D spectrum, using a technique introduced by Gerling (1992), in general into 3 or 4 wave systems (wind sea, swell, mixture (composition) of wind sea and swell and old wind sea). The wind sea systems are identified by comparing the phase velocity and direction of the spectral peak with the wind speed and direction. Each wave system is assumed to be generated by diferent physical events, and so are uncorrelated with each others, and each is represented by few parameters: SWH (or spectral energy), mean direction and mean frequency (see more details in VIOLANTE-CARVALHO et al., 2005). Each wave system of diferent spectra is cross-assigned with its counterpart—a wind sea system of a first-guess spectrum is correlated with the same system in the observed spectrum. If the wave system of the first-guess spectrum does not have a match in the observed spectrum it is superimposed on the analyzed spectrum. On the other hand, if the observed spectrum does not have a match in the first-guess spectrum it is superimposed on the first-guess spectrum. So a correspondence between all wave systems of the analyzed and observed spectra is reached and the wave systems that are cross-assigned are optimally interpolated generating an analyzed field of the parameters.

At this point the second step in the assimilation procedure can be implemented with the update of the spectrum and the correction of the wind field. The first-guess spectrum is rotated and rescaled to agree in direction, frequency and energy with the parameters derived from the interpolation and the new analyzed spectrum is created.

The wind is corrected using scaling power laws for a growing wind sea spectrum under quasi-equilibrium growth conditions (HASSELMANN et al., 1976). The wind field derived after the wave assimilation is interpolated with the first-guess wind yielding an updated wind field. The wave model can now be forced by the updated wind field and the diferences between the model and the SAR-retrieved wave spectrum are expected to be smaller.

A test run over a period of two months of assimilation of directional spectra extracted from ERS-1 SAR data is presented in Hasselmann et al. (1997). In that work the optimal interpolation scheme used by ECMWF in the assimilation of altimeter data was modified and applied to the assimilation of the full spectrum retrieved from SAR data. Another example of the application of OI is presented in the work of Voorrips et al. (1997), where wave parameters extracted from pitch-roll buoys in the North Sea are assimilated into a regional version of the WAM model. In Breivik et al. (1998) a routine for the assimilation of retrieved SAR spectra during a test period of 4 months was run parallel to the regular operation of the Norwegian Meteorological Institute (DNMI) second generation wave model using another sequential scheme, the Successive Corrections method. Successive Corrections is used operationally at DNMI so far assimilating only wave heights (BREIVIK and REISTAD, 1994). The OI has also been used operationally in several meteorological centers in their wave forecasting systems.

Since August 1993 ERS-1 altimeter wave height data have been assimilated by ECMWF into their WAM wave model (Lionello et al., 1992) while studies for the implementation of assimilation of the full directional SAR wave spectra are ongoing. Observations of SWH from the ERS-2 altimeter are assimilated using OI by the British Meteorological Oce (UKMO) in Bracknell into their second generation wave model (THOMAS, 1988; LORENC et al., 1991) and at the present the assimilation of retrieved SAR wave spectra is an active line of research in this center (James Gunson, personal communication).

On the whole the previous works have shown that the impact of assimilation of SAR spectra into wave models was very modest or neutral. The reason or reasons for this lack of improvement in the forecasting are not clear. One possibility is that the wave models have attained a level of sophistication where there is no clear improvement of the forecasting because of data assimilation. This seems unlikely to be the case.

Even third generation models such as WAM with state of the art representation of the physics of wave evolution have room for improvement, specially in the description of the low frequency part of the spectrum. The less well known wave dissipation source function causes a poorer representation of swell compared to the better description of the wind sea part of the spectrum (Komen et al., 1994). Another possible cause could be that methods to extract wave spectra from SAR images are not dealing properly with the complexities of the SAR imaging mechanisms and hence yielding poor retrievals (Violante-Carvalho et al., 2005). In addition it is not clear if the lack of improvement in the assimilation exercises are due to the assimilation schemes themselves or to the far fewer SAR observations (both in temporal and spatial coverage) compared to the number of model grid points.

However, besides the fact that these works have found no clear improvement in the forecasting, they have also used significant wave heights as independent data for the validation of the assimilation (VIOLANTE-CARVALHO and ROBINSON, 2004). The reason is that there is no other source of directional wave information over oceanic basins apart from SAR data. Most of the buoys deployed in the ocean measure only the surface elevation and hence only the frequency spectrum. The only source of wave information with coverage similar to SAR data is derived from altimeters, but the problem of using SWH to assess assimilation experiments is the averaging property of this parameter. More insights about the impact of the assimilation into the forecasting could be gained comparing the directional and spectral misfit between model and another source of directional wave information such as directional buoys. Another point worth mentioning is that the only new information added through the assimilation of SAR data is the the low frequency part of the spectrum since the wave model spectra are used to extend the spectral information beyond the high frequency cut-o . Therefore retrieval methods that do not rely on the wave model spectrum itself as first guess, like the cross-spectral method by Engen and Johnsen (1995), could bring more information to the assimilation procedure and hence improve the forecasting.

4 The Adjoint Technique

The basic idea of data assimilation in variational methods is to fit model predictions to observations by modifying the model input rather than the model output. The diferences between the model output and observations are measured by a cost function, and the assimilation is performed in order to minimize this cost function respecting the constraints of the model. The change in the wind field needed to generate a change in the wave field is determined by inverting the wave model equation, which has a very high computational cost specially for global operational implementation. The purpose of the Adjoint Method is to determine the minimum of the cost function without explicitly inverting the model equations, in such a way that the model equations and the adjoint model equations are solved in an iterative minimization loop.

Following the notation proposed by Komen et al. (1994) and describing the general data assimilation problem, the cost function *J* is constructed from three terms taking into account the diference between observed and modeled data J^d, the misfit between the model data and first-guess model values j^f and the diference between the control variables and first-guess control variables j^c (such the wind input and the initial wave field). J is a quadratic function that penalizes deviations of the model from observations and first-guesses, and its minimization yields the values of the control variables that make the model results fit best to the data and first-guesses available.

$$J = J^d + J^f + J^c \qquad (8)$$

Since J is positive definite, it is diferentiable and always has a point of minimum.

Then the variations with respect to the control variables $(\partial J / \partial c)$ is called the gradient of the cost and must be zero at its minimum.

The minimization of (8) is very time consuming in computational terms (actually the linear form of J is calculated by direct minimization) since the model data are implicit functions of the control variables. To avoid the direct inversion of the model equations, the Lagrange function L is constructed using the multipliers method

$$L = J + \lambda_i E_i \qquad (9)$$

by adding the Lagrange multiplier § times the models equations (in matrix representation) E to the cost function J (de las Heras, 1994; Komen et al., 1994).

Taking into consideration that the function L is odd its extremum has a stationary point that corresponds to the minimum of the cost function. More specificaly, the total derivative of J is the same as the partial derivative of L with respect to the control variables $(\partial L/\partial c)$ and both vanish at the point of minimum. This point can be determined by taking the partial derivatives of L with respect to all the arguments of the problem and setting the results to zero. The variation of the Lagrange function with respect toλ

$$\partial L / \partial \lambda_i = 0 \qquad (10)$$

yields the model equations, which can be solved forward in time. The derivative of with respect to the model data

$$\partial L / \partial x_i = 0 \qquad (11)$$

is called the adjoint of the wave model and can be solved backward in time. As has already been pointed out is the gradient of J or the cost-function gradient.

The problem of solving the model equations explicitly in order to compute the cost-function gradient is avoided by solving the linearized model equations (10—12) in an iterative way. As the gradient will be zero only for specific values of the control variables they are used to search the minimum of the cost. Choosing a first-guess for the control variables and solving (10) the solution is the model parameter of interest and the cost function J is determined. The adjoint model (11) can be solved backward in time to yield the value of □ and the gradient of J can be extracted using (12).

$$\partial L / \partial c = 0 \qquad (12)$$

If the value reached is not acceptable, the control variables can be updated and the procedure repeated until the minimum is approached. Multiple integrations of the model equations and the adjoint model equations are required, which can have a computationally expensive cost in particular for third-generation models in global runs.

The complication of deriving the adjoint model equations from the model equations was avoided by Hersbach (1998). In that work an adjoint model compiler was used to compute the code automatically line by line generating the adjoint of the full-dimensional WAM. The adjoint was used for inverse modeling with the object to get better estimates of several model parameters in the sink and source terms. In this way it is possible to determine whether misfits between model and data are caused by wrong wind inputs or by deficiencies in the model formulation (or in the linearization of the model equations). So far the adjoint method has been applied in wave data assimilation only to the simpler inverse modeling exercise, with the exception of de las Heras (1994) who worked with a one-dimensional version of the WAM to assimilate wave heights using synthetic data. For short period tests (of one day) the results obtained by de las Heras (1994) using the adjoint method were superior than the results from twin experiments using a simpler optimal interpolation method. For periods longer than one day the behavior of the gradient is more complex and the value of the minimum of the cost function was not attained in many cases.

5 The Green's Function Method

In the Adjoint Method the computationally expensive direct inversion of the model equations is avoided by solving the linearized model equations in an iterative way.

Despite that, this method still requires an order of magnitude more computer time than the integration of the wave model which seems to be very costly for global operational implementation. The Green's Function Method on the other hand also avoids the direct inversion of the model equations, but does so by relying on a number of physical approximations. The wave spectrum perturbations are expressed by the impulse response (or Green's) function over the wind field perturbations, and are inverted without the need of iterations, implying a computational time of the same order as the integration of the model.

The main assumption is that the wind perturbations that generate the spectrum perturbations are associated with a small region in space and time and therefore can be approximated by a -function. This hypothesis is intuitively plausible but it lacks mathematical rigor compared to the Adjoint Method. Once the wave component becomes swell the wind speed has no more influence on it, but on the other hand in the generation region its presence is important during the wave growth. The wave spectrum response to the wind input is to shift the spectral peak towards lower frequencies through nonlinear wave to wave interactions, transferring energy from the region just beyond the spectral peak to the region below the spectral peak which maintains a quasi-equilibrium spectral shape. Thus the impact of the wind is scattered over higher and lower frequencies through this stabilizing efect of the spectrum shape, being only retained and transported when the wave component leaves the generation area, that is as swell, propagating undisturbed. So the most sensitive region of the wave spectrum is the one that last received the input from the wind in the transition between wind sea and swell.

The assimilation scheme consists of minimizing the diferences between model data and observations through the following cost function (BAUER et al., 1996)

$$J = \sum_{r}^{n_{obs}} \left\{ \frac{(d_r^f - d_r^0 - d_r)^2}{(\sigma_r^d)^2} \right\} + C \sum_{p} \left\{ u_p^2 + v_p^2 \right\} \tag{13}$$

where *d* are the first-guess of the model data and the respective observed value, d_r is the modification after the optimization, u_p and v_p are the changes in the wind field in x and y components in a point p in space, n_{obs} is the number of observations, d_r is the standard deviation of the measurements and C is a weighting factor. The Green's Function Method computes the model modifications d_r which are correlated with the modifications in the wind input u_p and v_p respecting the constraints of the model.

In order to minimize the cost function, (13) must be expressed in terms of the control variables u_p and v_p, in a way that the response of the wave spectrum x described by the perturbations in the wind field u is expressed by the Green's function. In practice, the integration of the response function requires the inversion of the Green's function operator, which is not feasible due to the complexity of this matrix which involves the whole source function. Relying on the assumption that only a specific small region of the wind field causes a perturbation in a component of the wave spectrum, the Green's function can be approximated by a δ - function representing the relation between the wind changes (u_p, v_p) = [u(x_p, t_p), v(x_p, t_p)] in a point in the past (x_p, t_p) and the spectral energy changes in the observation point (x_r, t_r). The point (x_p, t_p) determines the influence point or the point of the last wind input that must be altered to yield the spectral modification in the component k and point x_r. The influence point (x_p, t_p) can be determined by tracing back the wave component using the wave age along the great circle path at its group velocity c_g. So far the Green's function assimilation method was run for synthetic wind cases (Bauer et al., 1996) with no rerun of the wave model in order to check the wave field corrections. A more realistic case was applied to determine the wind field corrections during a storm in the North Atlantic (BAUER et al., 1997). The results were compared with the Optimal Interpolation scheme and the wind corrections have a general good agreement, but again a comparison with the new model output generated by the updated wind field was not performed. Although quite attractive because it is less expensive in computational terms, the Green's function method relies on some simplifications, the strongest being about the localization of the wind region of influence. Perturbations in the wave spectrum are assumed to be caused by perturbations in the wind field in some specific region (in space and time) that are approximated by a -function, a rather unrealistic supposition.

In addition the corrections to the wind are estimated using observations available during one time level in general of 6 hours—like sequential methods—rather than over different times—like variational methods. However since the constraints of the model are maintained the wave spectrum computed after the assimilation is consistent with the model dynamics.

6 Discussion

Significant wave heights measured from satellite altimeters have so far been the most widely used information applied in wave models at several weather centers. However significant wave height is a mean parameter. Therefore a greater impact is expected on the wave analysis using techniques for the assimilation of the full two-dimensional spectrum due to the detailed spectral and directional information derived from this information. Global observations of directional spectra are now available with the SAR onboard ERS-1 and ERS-2 and more recently with the launch of ENVISAT carrying the Advanced Synthetic Aperture Radar (ASAR). This fact has opened up challenging possibilities and several studies are undergoing on how to best exploit this information to improve the wave forecasting. In the present work a comprehensive discussion of the theory of wave data assimilation is presented with a review of several assimilation studies developed in the last few years. Futhermore the techniques used so far in the assimilation of the two-dimensional spectrum are examined in more depth. Research in the area of wave data assimilation is in its early stages of development and implementation but some works have already indicated some exciting prospects for the future. One of the main issues of working with the assimilation of the two-dimensional spectrum is the high number of degrees of freedom involved in the problem. The approach of partitioning the spectrum into a number of wave systems each one represented by a set of parameters like mean direction of propagation, mean energy (SWH) and mean frequency seems reasonable and suits the assimilation problem very well. Another point that deserves to be mentioned is the calculation of the covariance matrices that requires long term statistics of the differences between observation and model. In the study by Voorrips et al. (1997) two years of buoy measurements in the North Sea were used to estimate the interpolation weights. However, specially due to the lack of long term observations, these interpolation weights are generally approximated by exponential expressions of the ratio between the distance model-observation and a correlation length scale. Thus an improvement of the estimation of the error covariance matrices necessarily requires several runs of the model to compute the statistical correlations.

A very particular characteristic of assimilating data into wave models which has no counterpart in meterorological or oceanic models is the distinction that must be imposed between wind sea and swell. Wind waves are very sensitive to the wind input which ensures that any correction of the wind sea part of the spectrum, if not accompanied by the respective correction in the wind input, reverts quickly to its original (incorrect) state. Therefore the correction of the wind sea part of the spectrum has only local influence. This became clear in the very first exercises on wave data assimilation which pointed out that combined wind and wave assimilation schemes, that is coupled wind-wave models, would be necessary for optimal assimilation purposes.

But the use at the present moment of coupled models is a very ambitious task and it seems that it will not be feasible, at least operationally, in the near future. The effect of swell corrections, on the other hand, can be felt over entire ocean basins over the period of several days. Once the wind waves leave the generation area, becoming swell, they propagate almost undisturbed and the analyzed (corrected) components will have a positive impact on the forecasting. In addition due to nonlinearities in the imaging processes only the low frequency part of the spectrum, before a high wavenumber cut-off, is directly mapped onto SAR images. As a consequence, if the main objective is to improve the wave forecasting rather than correct the wind input, it seems reasonable to assimilate only the swell part of the spectrum which is in the end the only information directly measured by SAR.

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