The soil available water estimate through the fractal theory

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Abstract

Numerous researches have applied the fractal geometry in the most diverse areas of scientific knowledge, having been shown very promising in the estimation of the physical properties of soil, water. In this sense, we aimed at this research estimate the water available in soil (AD) by the fractal theory based on two physical properties of easy determination; the water retention curve in the soil and the particle size distributionl. To implement the proposed model, parameters such as the fractal dimensions DSWRC and DPSD, fractals representative measure of pore size distribution of particles and soil, respectively, were defined as descriptive tools to estimate the curve of retention in the soil. Soil samples were collected at three depths 0-20, 20-40 and 40-60 cm, a total of 36 sampling points. The comparison of results with the model indicated that the proposed model is simple in its use and is able to predict satisfactorily both the retention curve and the available water in the soil.

Key words: physical soil; retention curve; irrigation management.

Introduction

The fractal geometry was introduced in the 70's, by Benoit Mandelbrot, Polish mathematician, who proposed through it, a way to represent better the complex structures and the irregular structures of nature. Unlike the Euclidean geometry, the fractal is presented as a tool effectively able to quantify and qualify series of temporal and spatial data describing, with greater accuracy, its complexity.

For the definition of Mandelbrot (1982) quoted by Reichardt and Timm (2004) and Hott (2005), the term fractal comes from the Latin *fractus*, which means fragmenting, breaking the parties, oppose the algebra term, which refers to the junction of the parties, that is, fractals are objects whose values of its dimensions are real numbers not intact, but the fractal dimension described by Mandelbrot is a measure of irregularity degree of the object seen in all scales of observation, where the fractal structure is the one in which parts of it are similar in all, that is, there are parts self-similar, statistically within the overall structure.

The theory of fractals has been widely used to describe the roughness of the land because of their efficiency in areas of sensitivity in different textures and is also useful in the classification of images definitions of landscapes' diversity, determination of operational scale of natural phenomena in digital images, effect on the conversion of data into geographical information systems, and in staggering applied to variability in space remote sensing for interference in the administration, evolution, ecology, resource sampling and simulation (LAUX; PEREIRA, 2005).

In soil science, the fractals are used to describe the route of infiltration and water's redistribution, in addition to modeling, efficiently, the occurrence of phenomena in these two cases, since the soil is a three-dimensional body.

The fractal dimension can be determined with some soil physical attributes, allowing studies on new approaches based on physical parameters, to occupy the studies' space using purely empirical parameters (HOTT et al., 2005). According to this, the main point of this study is to estimate the soil water available through the fractal dimension based on the retention curve and also based on the curve of size particle distribution.

Given the above this research has as objective to verify the adequacy of the fractal dimension method in the estimation of available water in the soil

Materials and methods

The research was conducted at the Laboratory of Soil Physics on Federal University of Campina Grande, which originally were determined the percentages of sand, silt and clay of the soil used on the survey which is, according to the classification of EMBRAPA (1999), a Neossolo Regolitico. Initially it was determined the curve of soil particles distribution and later, the retention curve

Soil samples were collected from an area cultivated with Sesame irrigated with a sprinkler system at depths of 0-20, 20-40 and 40-60 cm deep, in a total of 36 sampling points. In this research the soil water retention curve was determined in the laboratory using the Extractor of humidity of Reichardt, in the tensions of 10, 33, 100, 300, 500, 1000 and 1500 kPa, where the available water was obtained through the humidity corresponding to potential matrix of 10 and 1500 kPa, according to the methodology described by Guerra (2000) and the particle size distribution was determined by the method of hydrometer as recommended by Bouyoucos (1951).

One of the methodologies able to determine the soil fractal dimension is based on the average of soil solid particles size distribution once these value and fractal dimension are linked with the structure of the soil and can be estimated through the expression:

$$W(R)=cR^{\frac{3D^2-13D+14}{D^2-5D+4}} \qquad \text{(Equation 1)}$$

In which, W (R) - cumulative mass of soil particles, c - constant, R - average radius of soil particles,, D - fractal dimension.

The equation (1) is the expression through which it is estimated the percentage of the cumulative mass of particles, according to the radius of soil particles. In order to estimate D, it is applied the least squares method to obtain the coefficients of the linear straight, by means of consequence of the fractal dimension (D). The fractal dimension D, obtained by this procedure, results in the fractal dimension of surface (based on the size distribution of particles of soil) represented by DPSD.

Based on the equation proposed by Brooks and Corey (1964), modified by Pierrer et al. (1996) quoted by Andrade (2002), the process of non-linear regression to the tension data of water in the soil is applied. A more specific way between the pores volume and its radius, was found by Perrie et al. (1996) and is written as follows:

$$-\frac{dV(\leq r)}{dr} = \beta(E-D)\,r^{E-D-1} \qquad \text{(Equation 2)}$$

In which : E - represents the euclidiana dimension, β – a constant e r – pores radius.

To estimate the water content in soil, it is assumed that the value that the pores radius is inversely proportional to the potential hydraulic h, and h = A / r, in which A is a constant. Traditionally, it is used absolute values of tension of water in the soil, rather than negative tensions. The equation (2) is widely used to estimate the retention curve of water in the soil, even with other ways to correlate the soil pores with the water tension (TYLER and WHEATCRAFT, 1990; BIRD et al., 1996) .

The model to estimate the retention curve of water from the equation (2) presents a stiff relation with one law of physics, the water content of soil and tension with which the water is retained between the solid particles of soil (PIERRER et al., 1996). Based on the equation proposed by Brooks and Corey (1964), modified by Pierrer et al. (1996) quoted by Andrade (2002), the fractal dimension (DSWRC) was determined according to the process of non-linear regression to the data unit of soil (θ) versus the matrix potential of water in the soil (ψm) through the following expression:

$$\theta_{(h)} = \theta_r + (\theta_s - \theta_r)\left(\frac{h_s}{h}\right)^{3-D} \qquad \text{(Equation 3)}$$

In which: $\theta(h)$ the soil moisture based on volume, in the tension h, θ_S soil moisture on the basis of volume saturation, θ_R the residual soil moisture (corresponding to the point of permanent wilting), h_0 - absolute value of the water tension on the point of air entry and D represents the fractal dimension of distribution of the sizes of soil pores (DSWRC).

To the values of available water of the soil, descriptive statistics were applied to accurate and precise analysis of methods of interpolation, adopting as a standard, the standard error of estimate (RE). To fit the curve of water retention in the soil through the fractal dimension, a program in Java language was edited and to adjust the retention curve through the model of Brooks and Corey it was empirically used the computer program Soil Water Retention Curve (Version Beta 3.0).

Results and discussions

In table 1, we have described the statistic summary of the available water for three esteemed profundities of three models used on this research. In conformity with the table 1, the low value of the Pattern Deviation (PD) and the Coefficient of Variation (CV) indicates a variability reduction of the average values from the soil water proportion, in function of the original potential. The CV and PD (%) higher values were found in the depth of 40-60 cm. The DV had a notable increase to the humidity contents that corresponds to the biggest original potential of water in the soil (superior to 300 kPa), in the three profundities studied. According to MAYR and JARVIS (2000), the sandy soils are more sensitive to the variation in fractal dimension, D_{SWRC} and D_{PSD} values, <2 can indicate methodological problems on the obtainment of the retention curve.

It can be noted through table 1 that the smallest variations of the soil water proportion occurred in the presence of low tensions values applied in the soil, between the values of 0, 01 to 100 kPa and the highest variations between 300 to 1500 kPa to the highest potentials. It's also observed that for the four soil depths, the biggest water proportion variations of the soil occur in tension that corresponds to the permanent wizen point (1500 kPa). These results collaborate to those found out by Poulsen et al. (1999) that showed bigger variability, comprised between the water tension in the soil of 200 kPa and smaller variations between 1500 kPa. Figure 1, has a better visualization of the behavior of this variability.

It was verified that the Brooks and Corey models (1964) and BCD_{SWRC} had similar performance (Figure 1), esteemed humidity values next to the experimental values; but the BCD_{PSD} model

Table 1. Statistic summary of the average values of the volumetric humidity (cm3 cm-3) from the soil in the profundities studied in function to the original potential applied, Campina Grande, PB, 2006.

Ψ_m (kPa)	$\bar{X}$	DP	CV	$\bar{X}$	DP	CV	$\bar{X}$	DP	CV	$\bar{X}$	DP	CV
	------ observed-----			---- BCD_{SWRC} -----			-------- BCD_{PSD} --------			Broocks & Correy (1964)		
0-20 cm depth												
0.01	0.41	0.00	0.00	0.43	0.03	6.88	0.39	0.0008	0.0297	0.41	0.00	0.00
10	0.17	0.02	14.50	0.17	0.02	14.50	0.23	0.0012	0.0354	0.18	0.02	14.27
30	0.16	0.02	13.80	0.15	0.02	15.85	0.19	0.0002	0.0161	0.15	0.02	14.83
100	0.14	0.02	16.10	0.13	0.02	17.33	0.16	0.0004	0.0210	0.13	0.02	15.06
300	0.12	0.02	17.62	0.11	0.02	18.70	0.13	0.0001	0.0104	0.11	0.01	15.13
500	0.10	0.01	16.44	0.10	0.02	19.34	0.12	0.0001	0.0122	0.11	0.01	14.90
1000	0.09	0.01	16.35	0.09	0.02	20.20	0.11	0.0001	0.0123	0.10	0.01	14.87
1500	0.08	0.01	12.68	0.09	0.01	20.71	0.09	0.0008	0.0092	0.10	0.01	14.89
20-40 cm depth												
0.01	0.43	0.00	0.00	0.44	0.02	5.90	0.36	0.0007	0.027	0.43	0.00	0.00
10	0.17	0.01	9.24	0.17	0.01	9.24	0.23	0.00211	0.0459	0.18	0.01	8.09
30	0.16	0.01	7.03	0.15	0.01	9.96	0.20	0.0008	0.0293	0.15	0.01	8.20
100	0.14	0.01	8.29	0.12	0.01	10.79	0.17	0.00052	0.0229	0.12	0.01	8.24
300	0.12	0.01	10.07	0.11	0.01	11.56	0.18	0.00474	0.068	0.11	0.01	8.69
500	0.10	0.01	11.96	0.10	0.01	11.93	0.16	0.00371	0.0609	0.10	0.00	8.63
1000	0.09	0.01	8.21	0.09	0.01	12.43	0.12	0.00019	0.0139	0.10	0.00	9.05
1500	0.08	0.01	9.82	0.08	0.01	12.73	0.11	0.00033	0.0182	0.10	0.00	9.01
40-60 cm depth												
0.01	0.49	0.00	0.00	0.44	0.02	4.94	0.36	0.0009	0.030	0.49	0.00	0.00
10	0.18	0.05	31.58	0.14	0.02	18.24	0.24	0.00137	0.0371	0.19	0.06	35.45
30	0.15	0.05	31.57	0.12	0.02	20.61	0.21	0.00040	0.0202	0.14	0.044	30.01
100	0.13	0.04	35.12	0.10	0.02	23.24	0.19	0.00036	0.0191	0.11	0.03	27.35
300	0.10	0.03	29.75	0.08	0.02	25.67	0.17	0.00047	0.021	0.10	0.02	26.49
500	0.09	0.01	21.32	0.07	0.02	26.80	0.13	0.00022	0.0149	0.09	0.02	27.17
1000	0.08	0.01	23.40	0.07	0.02	28.35	0.11	0.00023	0.0154	0.09	0.02	26.47
1500	0.07	0.01	24.99	0.06	0.01	29.26	0.10	0.00011	0.0107	0.08	0.02	26.17

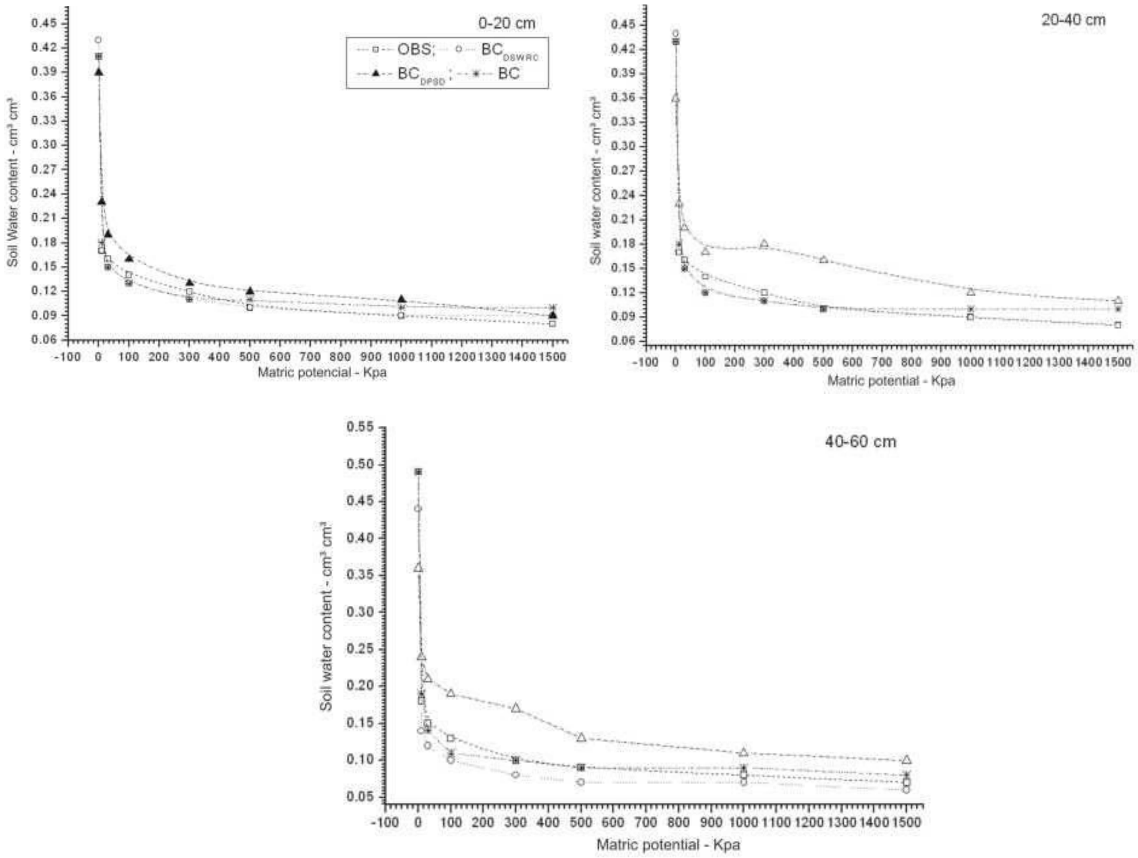
BCD_{SWRC} – with base in the soil water retention curve, BCD_{PSD} – with base in the particles size distribution $\bar{X}$ - Average, PD – pattern deviation e CV – Coefficient of variation

underestimated the humidity values when the soil was submitted to low tensions and overestimated when in high original potentials, indicating that for the study of the water movement in the soil, it is more suitable to consider the geometry and the medium diameter of the soil's pores (ANDRADE, 2002).

In the original potentials from 0 to 100 kPa, the BC and BCD_{SWRC} models showed themselves suitable with excellent estimative, but, the BC model underestimates the values of θ with the increase of the original potential, the BCD_{SWRC} model underestimates the values of θ in the interval from 0 to 300 kPa, estimating, however, with good precision the humidity content of the soil potential superior to 500 kPa (500kPa to 1500kPa), in the three profundities studied; so that, the curve of the θ values estimated by BCD_{SWRC} model was the one that was closer to the curve from the experimental data, so, giving better suitability, in relation to the other models tested (Figure 1). On the other hand, Tyler and Wheatcraft (1990) found good agreement of results between the retention curves estimated by fractal dimension of the distributions of particles and the one determined in the laboratory, mainly for soils that the value is D or bigger than 3.

In table 2, displays the parameters values of the average estimative standard error (ES) used to evaluate the quality and compare the performance of

Figure 1. Water retention curve in the soil adjusted by the three models evaluated, in the depths of 0-20, 20-40 and 40-60 cm, Campina Grande, PB, 2006.



Y = Soil water content X = matric potential the proposed model by Brooks and Corey modified by Pierrer in prediction of the soil water retention curve in relation to the average values.

Table 2 can be observed that the data adjustment by BCD_{PSD} model resulted in high values of 11.416; 23.275 and 15.336, for the three profundities, respectively; but with the BC and BCD_{SWRC} models these parameters were smaller in relation to the observed values (MC). These results showed that the BC and BCD_{SWRC} models adjusted better to the data of water content in the soil in function to the original potential [$\theta(\Psi)$], showing themselves satisfactory when compared to the observed data. That behavior is associated to fact that the D_{SWRC} variation has physics factors directly related to the soil water retention, as the pores sizes distribution and soil aggregates' size (CASTRIGNANÒ and STELLUTI, 1999). This result makes evident that besides being potential; the D_{SWRC} is another interest property of the soil's physics that shows itself as an efficient tool capable to qualify the soil water retention.

In table 3, we have described the statistic summary of the available water for three esteemed profundities of three models used on this research. The average values of available water for the three profundities are normally distributed, and an increasing coefficient of variation as an addition of depth, for the three models studied, but in an admissible zone for soils.

Table 2. Estimative standard error for the soil water retention curve's adjustment models in the three profundities studied, Campina Grande, PB, 2006.

Models	Estimative standard error		
	0-20 cm	**20-40 cm**	**40-60 cm**
BCD_{SWRC}	0.0136	0.0124	0.0435
BCD_{PSD}	11.416	23.275	15.336
Brooks e Corey (BC)	0.0107	0.0105	0.0420

Table 3. Statistic summary of available water established by laboratory method, esteemed by Brooks & Corey model, modified with fractal (DPSD e DSWRC) and without fractal dimension, for the three profundities.

Statistic parameters	Soil depth (cm)		
	0-20	**20-40**	**40-60**
Observed (OBS)			
Average	9.242	9.305	10.978
Pattern Deviation (%)	1.598	1.126	5.543
Coefficient of Variation (%)	17.293	12.098	50.496
BCD_{SWRC}			
Average	8.371	8.622	7.949
Pattern Deviation (%)	0.674	0.554	0.762
Coefficient of Variation (%)	8.048	6.421	9.587
Estimative standard error (ES)	0.804	0.946	1.586
BCD_{PSD}			
Average	3.754	2.117	1.951
Pattern Deviation (%)	0.545	0.196	0.663
Coefficient of Variation (%)	14.508	9.240	33.978
Estimative standard error (ES)	7.889	10.243	14.337
Brooks e Corey (BC)			
Average	8.525	8.538	10.325
Pattern Deviation (%)	1.289	0.852	5.636
Coefficient of Variation (%)	3.600	2.400	16.500
Estimative standard error (ES)	1.049	1.157	1.970

It is statistically evident that the values of available water presented small variability in the profundities studied of 0-20 and 40-40 with higher addition in the soil depth of 40-60 cm (50.49%), when established by laboratory method.

For the BCD_{SWRC} method and without using the Brooks and Corey fractal dimension (BC), it is noted low values of the statistics parameters coefficient of variation and pattern deviation for the three soil depths studied; however, for the Brooks and Corey model with dimension fractal based on particles distribution curve (BCD_{SWRC}), it's noted a higher variability, being higher in the last soil depth. That fact can be associated to a big variation of the percentage composition of the soil textural separate, soil structure variation, as well as the size of the sample.

It's possible to observe excellent values for the Brooks and Corey model with fractal approximation based on the soil water retention curve (BCD_{SWRC}), showing itself satisfactory when compared to the observed data; this way, the BCD_{SWRC} model, displays itself suitable for estimation of available water in the soil, with estimative standard error of 0.804; 0.946 and 1,586%, respectively, in the profundities of 0-20, 20-40 and 40-60 cm (Table 3). This approach can be used as another efficient tool to predict the available water in the soil too, through the functional relation of the fractal dimension with the Brooks and Corey models.

Conclusion

Based on the obtained results, it's established the following conclusion in the estimative of available water in the soil:

The fractal theory through the determination of the fractal dimension based on the soil water retention curve (D_{SWRC}) esteemed with excellent precision the available water values, followed by the Brooks and Corey model without fractal approximation.

The incorporation of the D_{SWRC} parameter in the Brooks and Corey model brought more precisely estimative of available water in the soil, followed by the Brooks and Corey model without fractal approximation.

Brooks and Corey proposed method; modified to the fractal dimension fixed from the laboratory measurement of the soil water retention curve (D_{SWRC}) underestimates, as a rule, the values of soil water content, mainly for original potentials closer to the saturation.

BCD_{SWRC} – with base in the soil water retention curve, BCD_{PSD} – with base in the particles size distribution $\bar{x}$ - Average, PD – pattern deviation e CV – Coefficient of variation.

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